Note From last day...

$$\gamma^2 = -\omega^2 \mu \varepsilon_c = -\omega^2 \mu \left(\varepsilon - j\frac{\sigma}{\omega}\right)$$

$$\vec{\nabla}^2 \vec{E}_s - \gamma^2 \vec{E}_s = 0$$

Plane waves: defined in a direction $\hat{n}$

$$\vec{E}_s = \vec{E}_0 e^{\pm \gamma \hat{n} \cdot \vec{r}} \qquad \gamma = \alpha + j\beta.$$

possibly complex constant.

Special case: $\hat{n} = \hat{z}$ $\quad \hat{n} \cdot \vec{r} = z$

$\vec{E}_0$ is real

$$\vec{E}(z,t) = \mathrm{Re}\left[\vec{E}_0 e^{-\alpha z} e^{-j\beta z} e^{j\omega t}\right]$$

$$= \vec{E}_0 e^{-\alpha z} \cos(\omega t - \beta z)$$

$$\vec{H}(z,t) = \vec{H}_0 e^{-\alpha z} \cos(\omega t - \beta z)$$

Show that $E_{sz} = 0$

$$\vec{\nabla} \times \vec{H}_s = j\omega \varepsilon_c \vec{E}_s$$

z component of this

$$\frac{\partial}{\partial x} H_{sy} - \frac{\partial}{\partial y} H_{sx} = 0$$

$$\Rightarrow j\omega \varepsilon_c E_{sz} = 0$$

$$\therefore \; E_{sz} = 0$$

Thus if $\hat{n} = \hat{z}$, $\vec{E}$ is $\perp$ to $\hat{n}$

Now to show that the magnetic feild is also $\perp$ to the $\hat{n}$

So: $\vec{E_o} = \hat{x} E_{ox} + \hat{y} E_{oy}$

$$\vec{\nabla} \times \vec{E_s} = -j\omega\mu \vec{H_s}$$

$$= \begin{bmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ E_{ox} e^{-\gamma z} & E_{oy} e^{-\gamma z} & 0 \end{bmatrix}$$

$$= -\gamma(-\hat{x} E_{oy} + \hat{y} E_{ox}) e^{-\gamma z}$$

$\perp$ to $\hat{n} = \hat{z}$

$$= -j\omega\mu \vec{H_s}$$

$\therefore$ $\vec{H_s}$ is $\perp$ to $\hat{n}$

$$\vec{H_s} = (\hat{x} H_{ox} + \hat{y} H_{oy}) e^{-\gamma z}$$ by comparing

$$\therefore \quad H_{ox} = \frac{-j}{\omega\mu}(-\gamma) E_{oy}$$

$$H_{oy} = \frac{j}{\omega\mu}(-\gamma) E_{ox}$$

$$\vec{H_s} \cdot \vec{E_s} = (\vec{E_o} \cdot \vec{H_o}) e^{-2\gamma z}$$

$$= 0$$

Summary $\quad \vec{E} \cdot \hat{n} = 0 \qquad \vec{H} \cdot \vec{E} = 0$

$\vec{H} \cdot \hat{n} = 0$

plane Waves

Solve for $\gamma = \alpha + j\beta$. in terms of μ, ε, σ.

$$\gamma^2 = -\omega^2 \mu \varepsilon \left(1 - j\frac{\sigma}{\omega}\right)$$

$$= (-1)\,\omega^2 \mu \varepsilon \left(1 - j\frac{\sigma}{\omega}\right)$$

$$\gamma = j\omega\sqrt{\mu\varepsilon}\left(1 - j\frac{\sigma}{\omega}\right)^{1/2}$$

$$\gamma = \alpha + j\beta \quad ; \quad \gamma^2 = \alpha^2 - \beta^2 + 2j\alpha\beta$$

$$\alpha^2 = \beta^2 = -\omega^2\mu\varepsilon$$

$$2j\alpha\beta = -j\omega\mu\varepsilon\sigma$$

$$\therefore \beta = \frac{\omega\varepsilon\sigma\mu}{2\alpha}$$

$$\alpha^2 - \beta^2 = \alpha^2 - \left(\frac{\omega^2\varepsilon^2\sigma^2\mu^2}{2\alpha^2}\right)$$

Oh... he fucked this right up... check the text.... plus these are given on cheat sheet.

$$\alpha = \sqrt{\frac{\omega^2\mu\varepsilon}{2}}\left[\left(1 + \frac{\sigma^2}{\omega^2\varepsilon^2}\right)^{1/2} - 1\right]$$

$$\beta = \sqrt{\frac{\omega^2\mu\varepsilon}{2}}\left[\left(1 + \frac{\sigma^2}{\omega^2\varepsilon^2}\right)^{1/2} + 1\right]$$

Note: $\sigma = 0$ then $\alpha = 0$
$\beta = \sqrt{\omega^2\mu\varepsilon}$

Assume: $\hat{n} = \hat{z}$

$$\vec{E_0} = \hat{x}\,|E_{0x}|\,e^{j\theta_x}$$

$$\vec{H_0} = \hat{y}\,|H_{0y}|\,e^{j\theta_y}$$

$$\vec{E}(z,t) = Re\left[\hat{x}|E_{ox}|e^{j\theta_x} e^{-\alpha z} e^{-j\beta z} e^{j\omega t}\right]$$

$$= \hat{x}|E_{ox}|e^{-\alpha z}\cos(\omega t - \beta z + \theta_x)$$

right traveling

$\delta = \frac{1}{\alpha}$: characteristic of how deep the wave propogates.

$$\vec{E}(z,t) = \hat{x}|E_{ox}|e^{\alpha z}\cos(\omega t + \beta z + \theta_x)$$

left traveling.

$$\mu = \pm\, \omega/\beta$$

wavelength: distance between two peaks assuming $\alpha = 0$.

$$\lambda \equiv \frac{2\pi}{\beta}$$

Units of $\vec{E}$: V/m

Units of $\vec{H}$ = Amp/m

$$\frac{|\vec{E}|}{|\vec{H}|} = \frac{V}{I}$$

note there is nothing to garentee that both vectors will reach their peak at the same time.

Power density: $|\vec{E}||\vec{H}|$